Continuity Equation Validation for Nonhomogeneous Traffic

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Abstract: The continuity equation involving traffic flow expresses the relationship between density, flow, and speed. Density equals the flow divided by space mean speed. The formulation of this equation in the year 1952 has two important assumptions. One assumption is that spacing and speed are constant, i.e., uncongested conditions with moderate to slightly high volumes. The other assumption is that homogeneous traffic prevails, vehicle composition is uniform and vehicles behave within strict lane discipline rules. To determine if the continuity equation is valid under nonhomogeneous traffic conditions, one performs an experiment involving data collection of density, flow, and speed at three midblock sites in India. Data collection occurred when uncongested conditions prevailed with moderate to slightly high volumes. Comparing the average density derived from observed densities in the field to the density derived from the continuity equation reveals whether or not the continuity equation accurately predicts average density under nonhomogeneous traffic conditions. Similar traffic operating characteristics served as the basis for grouping vehicles into five traffic entity types. The association between average density based on observed densities of nonhomogeneous traffic and density derived from the continuity equation had a correlation coefficient of +0.88 for light, four-wheeler type, +0.85 for heavy vehicle type, +0.90 for motorized, three-wheeler type, +0.83 for motorized, two-wheeler type, and +0.50 for nonmotorized, two- and three-wheeler type. Additionally, a nonparametric test, i.e., Wilcoxon signed-rank test, compared observed and derived densities. At a 95% confidence level, no significant difference existed between observed and derived densities of light four-wheeler type, heavy vehicle type, motorized two-wheeler type, nonmotorized two- and three-wheeler type, and between cumulative observed and cumulative derived density. Only in the case of the motorized three-wheeler type, the observed, and derived densities are significantly different. These moderate to strong correlations coupled with the results of the nonparametric test validate the application of the continuity equation when traffic is nonhomogeneous.

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Background

Homogeneous and Nonhomogeneous Traffic

The best way to illustrate the difference between homogeneous and nonhomogeneous traffic at nonintersection segments is to compare the photographs in Figs. 1 and 2. Homogeneous traffic has strict lane discipline and has traffic entity types whose physical dimensions do not vary much. In the United States, the *Highway Capacity Manual* (HCM 2000) mentions six traffic entity types: (1) passenger car includes vans; (2) heavy vehicle includes buses and trucks; (3) recreational vehicle; (4) motorcycle; (5) pedestrian; and (6) bicycle (*Highway Capacity Manual* 2000). HCM mentions motorcycles, but indicates that motorcycles are a small percentage of registered vehicles in the United States, i.e., "Motorcycles not included in total" (*Highway Capacity Manual* 2000). After that note, motorcycles are not included in any HCM methodologies.

In the nonhomogeneous traffic photograph of Fig. 2, loose-lane discipline prevails. Some may describe it as chaotic. The composition of nonhomogeneous traffic is more than just passenger cars and heavy vehicles. In addition to passenger cars, motorized two-wheelers, motorized three-wheelers, minitrucks, minibuses, bicycles, pedestrians, animals, animal-drawn carts, and vendor push-pull carts on the road are usually present in the composition. The physical dimensions of the traffic entities vary greatly. Operationally, acceleration and deceleration characteristics vary greatly because nonmotorized traffic entities exist along with motorized vehicles on the road. Further, the characteristics vary among motorized vehicle types with motorized two-wheelers that typically have 100 cc engines operating side by side with passenger cars that typically have 1,200 cc engines. A facility has nonhomogeneous traffic when its peak hour volume has less than 85% passenger cars and has less than 90% passenger cars and heavy vehicles (Fazio and Tiwari 1995). Verification of this working definition occurred at many Indian traffic facilities (Fazio and Tiwari 1995).

Many engineering practitioners are uncertain in using traffic operation and design models developed using homogeneous

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Fig. 1. Homogeneous traffic

Fig. 2. Nonhomogeneous traffic

traffic data to solve traffic problems where nonhomogeneous conditions prevail. The underlying traffic flow theory of nonhomogeneous traffic is very different. However, using proper scientific methods, one may discover specific commonalities existing between the two theories.

Continuity Equation for Homogeneous Traffic

The continuity equation of traffic flow for homogeneous traffic is (Gerlough and Huber 1975)

$$k = q/\bar{u}_s \tag{1}$$

where q=traffic flow across a lane or lanes (vehicles/h); $\bar{u}_s$=space mean speed (km/h) (mi/h); and k=traffic density in a lane or lanes (vehicles/km) (vehicles/mi). Eq. (1) assumes constant spacing and constant speed, i.e., an uncongested condition with moderate to slightly high volume. Another name for this equation is the fundamental identity or fundamental equation of traffic flow. Wardrop (1952) originally developed the equation. Gerlough and Huber (1975)justify the equation on the "analysis of units." Another assumption that Wardrop makes that many ignore is that the equation applies only to homogeneous traffic, i.e.,

> It follows that the density of this stream in space, that is to say, the number of vehicles per unit length [underline added] of road at any instant (the concentration), is given by $k_i = q_i/v_i \quad i=1,2,\ldots c\ldots$

For homogeneous traffic, concentration uses unit length per lane because traffic streams usually flow in orderly columns. Concentration across the entire highway width in one traffic direction becomes a matter of adding the individual concentrations in each lane that comprises the total highway width.

At the microscopic level, i.e., traffic entity trajectories from a time-space diagram, the equation for concentration is

$$k = \frac{\sum t_i}{dx\, dt} \tag{2}$$

where t_i=time traffic entity i travels distance dx; and dt=time interval in which one measures flow.

Validation Experiment

Continuity Equation for Nonhomogeneous Traffic

Because Wardrop developed the original continuity equation from homogeneous traffic characteristics, an adjustment must occur to reflect nonhomogeneous traffic characteristics yet still maintain the basic relationship in the original equation. Many entity types comprise nonhomogeneous traffic. Each type has an average concentration in the highway area. The validation of Eq. (3) uses nonhomogeneous traffic field data

$$\bar{k}_j = \frac{q_j/W}{\bar{u}_{s,j}} \tag{3}$$

where j=traffic entity type, e.g., 2=heavy vehicle, 3=motorized three-wheeler; $\bar{k}_j$=average number of traffic entities of type j per unit area of highway, e.g., motorized two-wheelers/(km m) or motorized three-wheelers/(mi ft); W=cross-sectional width for measuring flow (e.g., m or ft); flow q_j=number of traffic entities of type j crossing the cross-sectional line of width W during a time interval, e.g., nonmotorized two-wheelers/h; and speed $\bar{u}_{s,j}$=space mean speed of traffic entities of type j that completely traverse the length of the highway area (km/h or mi/h). An assumption is that W is constant throughout the highway segment for all traffic entity types.

This modified continuity equation also passes the "analysis of units." Eq. (3) is subject to the same assumption of constant spacing and speed in the original Wardrop's equation. Nonhomogeneous traffic data collection for validation must occur under uncongested conditions with moderate to slightly high volumes. Since all traffic entity types share the same highway area at any time instant, Eq. (4) holds

$$\bar{k}_{nt} = \sum_{j=1}^{N} \bar{k}_j \tag{4}$$

where k_{nt}=average number of nonhomogeneous traffic entities per unit area of highway, e.g., entities/(km m) or entities/(mi ft); and N=total number of entity types in the nonhomogeneous traffic stream. In this case, N=5 as explained below. Further, the sum of the traffic subflows of the individual traffic entity types comprising nonhomogeneous traffic is total nonhomogeneous traffic flow

$$\frac{q_{\mathrm{nt}}}{W} = \sum_{j=1}^{N} \frac{q_j}{W} \tag{5}$$

where q_{nt}=total nonhomogeneous traffic flow.

In the experiment, the grouping of traffic entities created five categories or types. Vehicles based on similar traffic operating characteristics are grouped into five types: (1) motorized four-wheelers (M4W) contains cars, vans, minivans, jeeps, and light pickup trucks; (2) heavy vehicles (HVs) include trucks, minitrucks, buses, and minibuses; (3) motorized three-wheelers (M3W) have autorickshaws, high capacity autorickshaws, and tempos; (4) motorized two-wheelers (M2W) contain motorcycles, motor scooters, and mopeds; and (5) nonmotorized vehicles (NMVs) include bicycles and cycle-rickshaws. An insignificant amount of pedestrians, animals, pushcarts, pull carts, and animal-drawn carts, i.e., nonmotorized traffic entities (NMEs), were present on the highway area. Their insignificant numbers precluded them from the experiment.

Goal

The goal is to check the validity of the continuity equation when nonhomogeneous traffic prevails. This involved a comparison. The comparison involved deriving the average density from densities observed on a midblock section to the density derived from flows and space mean speeds.

Methodology

Data Collection and Processing

Data collection involved three Indian, midblock sites using camcorder and video technologies and used the same procedures from a previous study (Fazio and Tiwari 1995). The midblock sites are Panchsheel, Defence Colony, and Sundar Nagar. Data reduction occurred in the laboratory under controlled conditions. A strategically positioned, stationary camcorder captured a 40 m (130 ft)–60 m (200 ft) long, highway section, i.e., dx. This allowed for the viewing of the entire highway width in one direction. The calibration of distances to the video monitor involved using a measuring wheel to mark length and width distances at the site. In the laboratory, the observers knew these distances by watching the video monitor and seeing the marked distances. The videotaping at the three sites occurred in a peak period. The highest, consecutive, 12, 5-min counts constituted the peak hour at each site. During the peak hour in which the camcorder operated, observers reported no traffic congestion at the three sites. In addition, the three midblock sites had no bus stops that would influence nonhomogeneous traffic flow.

In the laboratory, observers first watched the time-stamped, videotape to count traffic entities that traversed a cross-sectional line whose length was the site's unidirectional, highway width. Observers tallied their counts by traffic entity type every 5 min during the period, i.e., dt. Processing the count data into volumes revealed that they were moderate to slightly high as shown in Table 1. Also, at all three sites in the peak hour, passenger cars comprised less than 85% of the traffic, and cars and trucks comprised less than 90%; nonhomogeneous traffic prevailed at all three sites.

Second, the observers reviewed the videotape to sample traffic entity speeds through the section's length. Observers noted the travel time in which every fifth traffic entity traversed between 40 m (130 ft) and 60 m (200 ft). From this time and distance, the derivation of speed of the sampled entity occurred. For each traffic entity type during the 5-min interval, this sampling derived speeds measured across a distance. Taking the mean of traffic entity speeds produced the time mean speed for that traffic entity type.

After adjustment for nonhomogeneous traffic, the equation for space mean speed becomes Eq. (1)

$$\bar{u}_{s,j} = \frac{n_j L}{\sum_{i=1}^{n_j} t_{i,j}} \tag{6}$$

where i=ith traffic entity; j=traffic entity type; $\bar{u}_{s,j}$=space mean speed of traffic entity type j [km/h (mi/h)]; n_j=number of traffic entities in type j; t_{ij}=time it takes the ith traffic entity of type j to travel across the length of the highway area (s); and L or dx=longitudinal length of section of the highway area (m or ft). Eq. (6) is the equation used in deriving space mean speeds of each nonhomogeneous traffic entity type. An equivalent equation to calculate space mean speeds of nonhomogeneous traffic types is the harmonic mean equation

$$\bar{u}_{s,j} = \frac{1}{\frac{1}{n_j}\sum_{i=1}^{n_j} \frac{1}{u_{i,j}}} \tag{7}$$

where $u_{i,j}$=speed of the ith traffic entity in type j to travel across an area of highway [km/h (mi/h)]. An approximation for space mean speed from time mean speed for a particular time interval is

$$\bar{u}_{s,j} = \bar{u}_{t,j} - \frac{\sigma_{t,j}^2}{\bar{u}_{t,j}} \tag{8}$$

where $\bar{u}_{t,j}$=arithmetic mean speed of traffic entities comprising type j [km/h (mi/h)]; and $\sigma_{t,j}^2$=speed is the speed variance of traffic entities in type j [km^2/h^2 (mi^2/h^2)].

Third, the observers rewound the videotape to sample traffic-entity type densities on the highway area. Every 30 s, observers freeze framed the videotape and counted the traffic entities by type on the roadway area. The 30-s, sampling rate allowed different entities to occupy the highway area, i.e., the snapshots were independent of one another. From these sampled, instantaneous densities, one derived the average density by type during the same 5-min intervals where volume and speed observations occurred.

Relationship of Space Mean Speed between Nonhomogeneous Traffic and Individual Traffic Entity Types

The space mean speed of nonhomogeneous traffic is not simply the weighted space mean speed of the individual nonhomogeneous traffic entity types by flow. Substituting for $\bar{k}_{nt}$ in Eq. (4) produces

$$\frac{q_{nt}/W}{\bar{u}_{s,nt}} = \sum_{j=1}^{N} \bar{k}_j \tag{9}$$

Solving for $\bar{u}_{s,nt}$

$$\bar{u}_{s,nt} = \frac{q_{nt}/W}{\sum_{j=1}^{N} \bar{k}_j} \tag{10}$$

Substituting $\bar{k}_j$ with Eq. (3)

$$\bar{u}_{s,nt} = \frac{q_{nt}/W}{\sum_{j=1}^{N} \frac{q_j/W}{\bar{u}_{s,j}}} = \frac{q_{nt}}{\sum \frac{q_j}{\bar{u}_{s,j}}} \tag{11}$$

Expressing nonhomogeneous traffic as a percent composition of total flow

Table 1. Nonhomogeneous Traffic Counts

Site name [length (width)]	5-min interval	Count, traffic entities					
		$T1$[a]	$T2$	$T3$	$T4$	$T5$	Total
Panchsheel [60.12 m (13.85 m)]	1	199	14	42	185	26	466
	2	207	16	45	184	25	477
	3	213	10	57	156	18	454
	4	227	16	58	187	19	507
	5	222	14	42	199	22	499
	6	179	9	40	139	16	383
	7	215	5	55	126	18	419
	8	216	9	47	130	7	409
	9	214	5	49	110	12	390
	10	180	7	47	109	9	352
	11	209	13	40	133	4	399
	12	209	10	48	121	11	399
Defence Colony [42.08 m (10.62 m)]	1	96	8	20	124	63	311
	2	93	7	22	122	72	316
	3	76	9	32	92	56	265
	4	96	5	31	106	43	281
	5	81	9	21	121	46	278
	6	102	5	25	110	56	298
	7	79	4	32	97	49	261
	8	74	6	31	104	40	255
	9	82	12	31	102	32	259
	10	85	8	33	85	40	251
	11	79	10	24	84	28	225
	12	88	5	21	64	31	209
Sundar Nagar [60.77 m (10.16 m)]	1	55	18	20	62	28	183
	2	81	10	21	81	25	218
	3	46	12	15	79	20	172
	4	70	6	18	78	20	192
	5	50	9	13	82	23	177
	6	58	10	18	61	24	171
	7	101	14	29	101	25	270
	8	86	11	22	105	23	247
	9	93	9	21	84	17	224
	10	88	12	22	83	12	217
	11	114	7	24	75	17	237
	12	94	9	21	86	11	221

[a]$T1$=Type 1=cars/vans/jeeps; $T2$=Type 2=trucks/buses/minibuses; $T3$=Type 3=motorized three-wheeler; $T4$=Type 4=motorized two-wheeler; and $T5$=Type 5=nonmotorized vehicle.

$$\bar{u}_{s,\text{nt}} = \frac{100}{\sum_{j=1}^{N} \frac{\%_j}{\bar{u}_{s,j}}} \tag{12}$$

where $\%_j$=traffic composition percent of traffic entity type j. Thus, the space mean speed of nonhomogeneous traffic is the weighted, harmonic space mean speed of the individual nonhomogeneous traffic entity type by flow.

Data Summary

Table 1 presents traffic count data by site, entity type, and time interval. Table 2 has the time mean speed data summary. The results of spot speed variance calculations are in Table 3. Within each traffic entity type, speed does not vary much; the coefficients of variation were usually less than 0.3. Table 4 shows the space mean speeds derived directly from data using Eq. (6). The summarization of the density data based upon videotape observations is in Table 5. Table 5 shows average, 30-s, sampled densities. Using the nonhomogeneous traffic continuity equation of Eq. (3), the resultant traffic concentrations appear in Table 6. Comparing the traffic concentrations in Table 5 to those in Table 6 produced measures of association by traffic entity type and by site.

Error Sources

Human errors in this traffic experiment involved counting errors in deriving volume and density and errors measuring travel time. Videotaped data allowed the luxury to spot check the counts and measurements. When miscounting or mismeasurement occurred, reobservation of the entire 5-min segment occurred. In this manner, the lessening of human errors occurred.

Hardware errors primarily involved the view of the elevated camcorder on a tripod. Because camcorders convert three dimen-

Table 2. Nonhomogeneous Traffic Spot Speeds

Site name [length (width)]	5-min interval	Time mean speed (km/h)				
		*T*1[a]	*T*2	*T*3	*T*4	*T*5
Panchsheel [60.12 m (13.85 m)]	1	24.6	18.5	22.0	25.3	7.1
	2	25.2	14.5	22.7	26.7	9.6
	3	25.2	10.4	24.7	25.2	7.9
	4	21.9	10.5	21.7	27.0	8.5
	5	23.4	13.5	20.4	26.0	10.2
	6	24.6	13.6	21.4	26.6	10.8
	7	24.7	16.6	20.8	28.6	7.3
	8	21.4	13.3	17.7	25.3	8.8
	9	22.8	10.8	18.9	26.0	7.2
	10	22.1	11.1	20.6	23.2	9.0
	11	22.4	12.4	19.5	24.5	9.1
	12	22.2	17.0	22.0	25.2	8.4
	COV[b]	0.1	0.4	0.2	0.2	0.2
Defence Colony [42.08 m (10.62 m)]	1	19.0	18.6	18.5	24.8	9.0
	2	30.5	25.0	24.0	26.7	8.7
	3	28.9	23.2	20.9	29.2	8.3
	4	28.6	21.2	22.4	29.6	12.2
	5	32.4	25.5	23.0	33.0	9.6
	6	36.2	26.5	27.2	31.2	9.6
	7	30.5	21.0	24.9	30.0	10.1
	8	29.9	27.6	25.5	28.7	8.8
	9	29.5	22.8	23.1	33.4	9.8
	10	29.3	22.6	24.4	32.3	8.7
	11	38.3	22.5	28.2	31.4	9.0
	12	29.2	18.5	24.0	37.5	9.7
	COV	0.3	0.2	0.2	0.2	0.3
Sundar Nagar [60.77 m (10.16 m)]	1	21.2	12.1	15.0	11.7	12.9
	2	19.3	13.7	15.7	12.2	7.3
	3	18.0	12.5	15.6	11.8	—
	4	19.9	14.0	13.4	12.6	4.1
	5	18.6	12.4	14.0	11.0	4.9
	6	20.0	14.1	15.4	12.0	—
	7	16.9	14.3	15.1	11.7	—
	8	17.5	15.8	13.7	11.7	—
	9	18.1	15.0	14.1	12.3	—
	10	16.8	16.2	15.2	11.5	—
	11	15.9	13.5	14.6	11.5	—
	12	17.0	14.7	13.1	11.1	4.9
	COV	0.2	0.3	0.1	0.5	0.6

[a]*T*1=Type 1=cars/vans/jeeps; *T*2=Type 2=trucks/buses/minibuses; *T*3=Type 3=motorized three-wheeler; *T*4=Type 4=motorized two-wheeler; and *T*5=Type 5=nonmotorized vehicle.

[b]COV=coefficient of variation.

sions into two dimensions for display on a video monitor, foreshortening became a limitation on measuring densities and speeds. Ideally, one should derive space mean speeds across a long distance, i.e., 0.5 km (1,650 ft). Sixty m (200 ft) was the camcorder's limitation because observers could clearly distinguish a length that far. At any greater length, observers could not distinguish between the distance marks. A speed study showed less than 1.5 km/h (0.9 mi/h) difference between space mean speeds and time mean speeds, i.e., measured across a long length or at a point, respectively (Garber and Hoel 2002). Measuring traffic entity speeds with portable loop detector technology or video imaging technology proved unfeasible under nonhomogeneous traffic conditions; loose lane discipline causes many traffic entities to bypass the loop detector in a delineated lane and to occupy simultaneously the loop area.

Plotting the difference between densities derived using the continuity equation and observed densities on the vertical axis with observed densities on the horizontal axis showed an error trend. Although the density difference points have approximately the same number above and below zero, an inverse, linear relationship exists. As observed density increases, the density difference decreases linearly. This relationship has more prominence for traffic entity types of slower average speeds such as nonmo-

Table 3. Nonhomogeneous Traffic Speed Variances

Site name [length (width)]	5-min interval	Speed variance (km^2/h^2)				
		$T1$[a]	$T2$	$T3$	$T4$	$T5$
Panchsheel [60.12 m (13.85 m)]	1	4.34	49.45	41.39	16.79	0.69
	2	5.12	72.47	9.31	52.11	3.58
	3	5.12	5.52	18.74	10.55	0.33
	4	6.76	15.70	18.41	74.71	0.84
	5	6.31	6.78	9.22	19.58	12.51
	6	15.67	83.56	11.84	59.34	0.59
	7	18.74	49.97	4.43	17.75	0.96
	8	19.11	17.00	18.43	36.00	4.03
	9	1.73	22.44	11.45	6.45	—
	10	18.28	1.89	7.47	15.24	—
	11	5.80	24.59	3.13	34.14	0.41
	12	7.16	68.69	10.22	10.55	0.09
Defence Colony [42.08 m (10.62 m)]	1	65.79	15.10	16.68	22.98	8.60
	2	42.97	13.01	3.47	32.98	3.91
	3	12.49	31.54	22.25	21.46	3.78
	4	65.57	18.68	5.99	36.53	31.03
	5	103.06	7.63	7.07	79.05	6.18
	6	62.90	47.81	13.46	100.28	5.14
	7	42.97	1.83	10.15	64.01	7.45
	8	21.46	21.94	7.63	30.31	8.99
	9	4.25	38.26	43.71	79.05	8.42
	10	46.58	11.23	14.88	17.21	7.82
	11	97.96	9.76	26.56	39.31	3.81
	12	57.64	28.43	19.12	83.29	15.08
Sundar Nagar [60.77 m (10.16 m)]	1	3.39	40.46	1.80	31.69	—
	2	15.55	5.51	0.51	52.57	—
	3	8.40	12.83	2.33	38.48	—
	4	9.28	18.42	1.92	48.70	—
	5	13.17	28.19	4.77	30.08	—
	6	12.46	2.99	2.49	35.91	—
	7	11.94	28.57	2.93	32.51	—
	8	3.51	8.14	0.79	33.81	—
	9	6.00	37.65	0.52	33.80	—
	10	2.30	12.71	2.34	36.02	—
	11	2.70	42.40	0.38	30.64	—
	12	3.85	25.16	1.53	28.55	—

[a]$T1$=Type 1=cars/vans/jeeps; $T2$=Type 2=trucks/buses/minibuses; $T3$=Type 3=motorized three-wheeler; $T4$=Type 4=motorized two-wheeler; and $T5$=Type 5=nonmotorized vehicle.

torized vehicles. The exact cause of this relationship requires further study, but traffic entity types using the street area differently is an underlying factor.

The methodology used in measuring instantaneous speed is another source of error. The calculations of space mean speed and density propagate this error because both values involved a portion of traffic entity length in their derivations. To calculate the instantaneous speed, one records the time required for the vehicle to cross the lines drawn on the screen from the front bumper touching the first line to the front bumper touching the second line. Dividing the distance between two lines drawn on the screen by the time taken to cross the line produces an approximate instantaneous speed at which the vehicle is traveling. Videotape technology on which one records or plays data uses a minimum movement of four frames at a time in the forward or backward direction. One frame has duration of 1/100 s. Therefore, four frames have duration of 0.04 s. When one plays the videotape, the minimum distance that the vehicle moves on the screen is equivalent to four frames, which is equal to the product of the instantaneous speed of the vehicle as shown by its own speedometer on the screen and 0.04 s. This creates a problem because the front bumper or rear bumper of the vehicle under consideration may not always exactly coincide with the marked line on the screen. The reason for this is that the distance between the front or rear bumper with the marked line is less than the distance that vehicle moves because of its own speed in four frames or 0.04 s. Fig. 3 illustrates this error.

The time one generally estimates is either more or less than the actual time taken by the vehicle to traverse the distance between the two lines on the screen. This introduces an error in the calculation of instantaneous speed, which is either more or less than the actual instantaneous speed of the vehicle. Figs. 3(a and b) together are an example of overestimation. Figs. 3(c and d) together are an example of underestimation.

Table 4. Nonhomogeneous Space Mean Speeds

Site name [length (width)]	5-min interval	Space mean speed (km/h) $T1^{a}$	$T2$	$T3$	$T4$	$T5$	Weighted harmonic
Panchsheel [60.12 m (13.85 m)]	1	24.5	15.1	20.3	24.5	7.1	20.9
	2	25.0	11.5	22.4	26.0	9.3	22.2
	3	25.0	10.0	24.0	25.5	7.8	22.3
	4	21.6	8.4	20.9	26.0	8.4	20.6
	5	23.2	13.2	20.0	27.1	9.6	22.3
	6	24.0	10.5	20.9	26.5	10.8	22.6
	7	24.0	14.9	20.6	28.9	7.2	22.3
	8	20.6	12.6	16.9	26.0	8.3	20.6
	9	22.8	8.7	18.3	26.5	7.2	21.1
	10	21.3	11.0	20.3	22.8	9.0	20.5
	11	22.2	11.1	19.4	24.5	9.1	21.5
	12	21.9	13.7	21.6	25.5	8.4	21.5
Defence Colony [42.08 m (10.62 m)]	1	15.7	17.8	17.8	23.9	8.0	15.0
	2	29.3	24.6	23.9	26.7	8.2	17.9
	3	28.4	22.2	19.8	29.3	7.9	17.7
	4	26.7	20.5	22.2	30.3	10.5	21.9
	5	30.3	25.2	22.7	33.7	8.9	21.8
	6	35.0	25.2	26.7	29.3	9.2	21.4
	7	29.3	20.9	24.6	29.3	9.5	20.6
	8	29.3	26.7	25.2	28.4	7.7	19.8
	9	29.3	21.1	20.7	33.7	8.8	22.4
	10	27.5	22.2	23.9	33.7	7.7	20.1
	11	36.4	22.2	27.5	31.3	8.5	23.7
	12	27.5	17.2	23.3	37.9	7.2	20.1
Sundar Nagar [60.77 m (10.16 m)]	1	21.1	9.0	14.9	9.0	12.9	12.2
	2	17.5	13.3	15.6	8.6	7.3	11.2
	3	17.9	11.1	15.4	8.7	—	—
	4	20.1	11.9	13.3	9.2	4.1	10.2
	5	17.9	10.3	13.7	8.7	4.9	9.4
	6	17.7	14.0	15.3	9.0	—	—
	7	16.7	12.2	14.9	8.9	2.7	9.1
	8	16.2	15.4	13.7	8.9	2.7	8.8
	9	17.8	12.5	14.1	9.1	—	—
	10	13.8	15.3	15.1	8.3	—	—
	11	15.8	9.9	14.6	8.8	—	—
	12	16.8	9.4	13.0	8.8	4.9	10.9

[a]$T1$=Type 1=cars/vans/jeeps; $T2$=Type 2=trucks/buses/minibuses; $T3$=Type 3=motorized three-wheeler; $T4$=Type 4=motorized two-wheeler; and $T5$=Type 5=nonmotorized vehicle.

Figs. 3(a and b) and Figs. 3(c and d) in pairs denote the approach and departure of the same vehicle. Both pairs are cases of extreme measurements.

One could quantify the maximum underestimation or maximum overestimation of speed possible in this experiment. The speed one obtains in the laboratory is between these two speeds. Figs. 3(a and b) give the lower limit underestimation, which is equal to

$$u_{\text{calculated}} = \frac{L}{2 + \left[\dfrac{L}{0.04 \times u_{\text{Speedometer}}}\right]} \tag{13}$$

where $[x]$=greatest integer less than or equal to x; L=distance in meters or feet between the two line; and $u_{\text{speedometer}}$=vehicle speed in meters per second or feet per second as shown by the speedometer of the vehicle. Figs. 3(c and d) represent the upper limit of overestimation

$$u_{\text{calculated}} = \frac{L}{\left[\dfrac{L}{0.04 \times u_{\text{speedometer}}}\right]} \tag{14}$$

where $[x]$=greatest integer less than or equal to x; L=distance in meters or feet between the two line on the screen; and $u_{\text{speedometer}}$=speed of the vehicle in meters per second or feet per second, as shown by the speedometer of the vehicle.

One tries minimizing the error in speed estimation by using the combination of Figs. 3(a and d) and Figs. 3(b and c). The minimization of instantaneous speed error occurs in this experiment because individual vehicle speed derivation used 40 m (130 ft)–60 m (200 ft) lengths.

Table 5. Observed Nonhomogeneous Traffic Concentration

Site name [length (width)]	5-min interval	Observed density [entities/(km m)]					
		$T1$[a]	$T2$	$T3$	$T4$	$T5$	Total
Panchsheel [60.12 m (13.85 m)]	1	4.5	1.1	0.4	5.4	5.4	16.8
	2	5.1	0.9	2.5	4.3	7.2	19.9
	3	5.6	0.7	1.6	4.5	6.9	19.2
	4	3.6	2.2	2.5	5.4	6.7	20.4
	5	4.5	0.4	0.3	5.8	6.7	17.7
	6	6.0	0.2	1.8	6.9	8.5	23.5
	7	5.1	0.7	1.8	5.6	4.5	17.7
	8	5.4	0.7	2.0	5.6	4.0	17.7
	9	4.7	0.7	2.5	6.7	5.1	19.7
	10	6.9	0.4	2.5	3.4	4.0	17.2
	11	3.6	0.4	2.5	3.8	3.6	13.9
	12	5.4	0.9	1.3	4.3	3.8	15.7
Defence Colony [42.08 m (10.62 m)]	1	7.6	1.4	0.9	3.8	1.3	15.0
	2	7.8	1.2	1.9	6.5	2.5	19.8
	3	5.5	0.9	3.4	4.7	1.1	15.7
	4	8.0	1.3	3.9	6.1	1.2	20.5
	5	6.5	1.8	3.1	7.4	1.8	20.5
	6	5.3	1.1	3.8	8.4	2.7	21.2
	7	5.5	0.4	3.1	5.3	1.1	15.3
	8	7.9	0.6	2.1	4.2	1.4	16.3
	9	6.6	0.9	4.0	5.3	1.5	18.4
	10	7.3	1.1	3.2	4.2	1.3	17.1
	11	5.5	1.2	2.5	3.3	0.7	13.2
	12	7.1	0.7	2.4	3.9	0.4	14.4
Sundar Nagar [60.77 m (10.16 m)]	1	4.5	1.5	2.8	3.5	3.2	15.4
	2	6.1	0.9	1.9	5.1	2.3	16.3
	3	3.5	1.3	1.6	6.7	2.9	16.0
	4	4.2	0.6	1.6	5.2	0.7	12.4
	5	5.2	0.9	1.2	5.8	3.2	16.3
	6	2.3	0.4	0.6	5.5	2.8	11.6
	7	4.2	0.6	2.6	6.5	2.0	17.0
	8	5.2	1.2	2.6	7.3	3.8	20.1
	9	4.1	0.9	1.9	7.6	2.8	17.2
	10	5.7	1.3	2.2	7.1	1.3	17.6
	11	6.1	0.6	2.5	5.5	1.3	16.0
	12	6.3	0.4	2.3	7.3	2.0	18.3

[a]$T1$=Type 1=cars/vans/jeeps; $T2$=Type 2=trucks/buses/minibuses; $T3$=Type 3=motorized three-wheeler; $T4$=Type 4=motorized two-wheeler; and $T5$=Type 5=nonmotorized vehicle.

Another source of error can be the sampling rate selected for observed density that was every 30 s. Using such a discrete sampling rate provided an average density that only estimates the concentration of Eq. (2) that is continuum based. Using a finer sampling rate would better approximate Eq. (2) density. This study uses a rectangular time-space area of $dt=5$ min by $dx=40$ m (130 ft)–60 m (200 ft). However, the sampling rate should be large enough so that one does not count traffic entities more than once. This rate guarantees that observations are independent. The slowest vehicle types were nonmotorized two-wheeler and three-wheeler with an average speed of 7 km/h (4 mi/h). The slowest vehicle would require approximately 30 s traversing 60 m (200 ft). Motorized vehicle speeds varied between 20 km/h (12 mi/h) and 40 km/h (25 mi/h) so none were counted more than once. If the vehicles are not randomly distributed then the selected sample may represent biased data. This is especially applicable to vehicle types that have low volume shares in total vehicular flow and driving characteristics that lead to platoon formation.

Results

Correlation

The association between observed and derived densities given 36 observations for each traffic entity type had correlation coefficients with zero intercept of +0.89 for nonheavy four-wheel vehicles, +0.85 for heavy vehicles, +0.90 for motorized three-wheel vehicles, +0.83 for motorized two-wheel vehicles, and +0.50 for nonmotorized vehicles. Given all 180 observations, the correlation coefficient with no intercept is +0.78 when combining all

Table 6. Continuity Equation Derived Nonhomogeneous Traffic Concentration

Site name [length (width)]	5-min interval	Derived density [entities/(km m)]					
		*T*1[a]	*T*2	*T*3	*T*4	*T*5	Total
Panchsheel [60.12 m (13.85 m)]	1	7.0	0.8	1.8	6.5	3.2	19.4
	2	7.2	1.2	1.7	6.1	2.3	18.6
	3	7.4	0.9	2.1	5.3	2.0	17.6
	4	9.1	1.6	2.4	6.2	2.0	21.3
	5	8.3	0.9	1.8	6.4	2.0	19.4
	6	6.4	0.7	1.7	4.5	1.3	14.7
	7	7.7	0.3	2.3	3.8	2.2	16.3
	8	9.1	0.6	2.4	4.3	0.7	17.2
	9	8.1	0.5	2.3	3.6	1.4	16.0
	10	7.3	0.6	2.0	4.1	0.9	14.9
	11	8.2	1.0	1.8	4.7	0.4	16.0
	12	8.3	0.6	1.9	4.1	1.1	16.1
Defence Colony [42.08 m (10.62 m)]	1	6.9	0.5	1.3	5.9	8.9	23.4
	2	3.6	0.3	1.0	5.2	9.9	20.0
	3	3.0	0.5	1.8	3.5	8.0	16.9
	4	4.1	0.3	1.6	4.0	4.6	14.5
	5	3.0	0.4	1.0	4.1	5.9	14.4
	6	3.3	0.2	1.1	4.2	6.9	15.7
	7	3.0	0.2	1.5	3.7	5.8	14.3
	8	2.9	0.3	1.4	4.1	5.9	14.5
	9	3.2	0.6	1.7	3.4	4.1	13.0
	10	3.5	0.4	1.6	2.9	5.8	14.1
	11	2.5	0.5	1.0	3.0	3.7	10.7
	12	3.6	0.3	1.0	1.9	4.9	11.7
Sundar Nagar [60.77 m (10.16 m)]	1	3.1	2.4	1.6	8.2	2.6	17.8
	2	5.5	0.9	1.6	11.1	4.0	23.1
	3	3.0	1.3	1.1	10.7	—	16.2
	4	4.1	0.6	1.6	10.0	5.8	22.1
	5	3.3	1.0	1.1	11.2	5.6	22.2
	6	3.9	0.8	1.4	8.0	—	14.2
	7	7.1	1.4	2.3	13.5	10.9	35.2
	8	6.3	0.8	1.9	14.0	10.2	33.2
	9	6.2	0.9	1.8	10.9	—	19.7
	10	7.5	0.9	1.7	11.8	—	21.9
	11	8.5	0.8	1.9	10.1	—	21.4
	12	6.6	1.1	1.9	11.6	2.7	23.9

[a]*T*1=Type 1=cars/vans/jeeps; *T*2=Type 2=trucks/buses/minibuses; *T*3=Type 3=motorized three-wheeler; *T*4=Type 4=motorized two-wheeler; and *T*5=Type 5=nonmotorized vehicle.

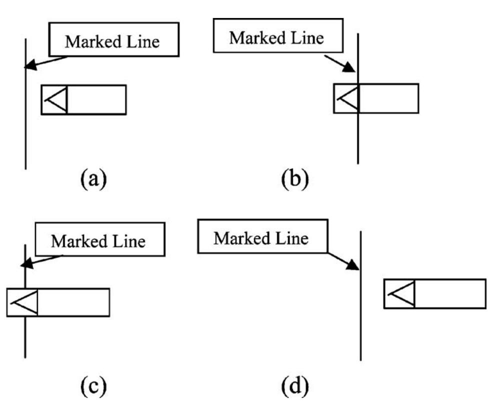

Fig. 3. Traffic entity speed measurement error

traffic vehicle types. Given the variability that exists in a traffic stream due to driver behavior, a correlation coefficient of +0.70 or more represents a strong correlation. Considering the correlation coefficients by site, 60 observations for each site revealed that the density association with no intercept for Panchsheel was +0.84, for Defence Colony +0.73, and for Sundar Nagar +0.77.

All density associations by vehicle type were higher than +0.83 except for the nonmotorized vehicles at +0.50. A possible reason for a moderate correlation for nonmotorized vehicles is that they use the highway area differently from motorized vehicles; they concentrate in a relatively narrow highway width along the road edge. Continuity equation, i.e., Eq. (3), usage proved valid in the three Indian sites where nonhomogeneous traffic prevailed.

It is tempting to compare observation-based $\bar{k}_{nt}$ to derive $\bar{k}_{nt}$. However, the speeds in Table 4 indicate that, although speeds remain relatively constant within each traffic entity type, they are

Table 7. Wilcoxon Signed Rank Test *P* Values

Vehicle type	*P* values
Light four-wheeler type	0.937
Heavy vehicle type	0.057
Motorized three-wheeler type	0.003
Motorized two-wheeler type	0.111
Nonmotorized two- and three-wheeler type	0.196
Cumulative of all vehicles	0.604

not constant between different types of vehicles. A large average speed difference exists between nonmotorized and motorized vehicle types. As mentioned earlier, one assumption regarding the use of the Wardrop equation is that vehicular speeds are relatively constant, i.e., low coefficient of variation. The nonhomogeneous traffic density $\bar{k}_{nt}$ includes both motorized and nonmotorized vehicle types whose average speeds are greatly different. A comparison between observed and predicted $\bar{k}_{nt}$ does not meet that assumption.

Wilcoxon Signed-Rank Test between Observed Density and Derived Density

A nonparametric test, i.e., the Wilcoxon signed-rank test, compared observed and derived densities. The Wilcoxon signed rank test does not make any distributional assumption of the data. At a 95%, confidence level, no significant difference existed between observed and derived densities of light four-wheeler type, heavy vehicle type, motorized two-wheeler type, nonmotorized two- and three-wheeler type, and cumulative density. Only in the case of the motorized three-wheeler type were the observed and derived densities significantly different. In this comparison, speeds are relatively constant in the intratraffic types.

One rejects the null hypothesis that the observed and calculated densities are significantly different when the p values from Wilcoxon signed rank test are greater than 0.05. Table 7 shows the p values for the two-tailed Wilcoxon signed rank test. One rejects the null hypothesis for all vehicle types except for the motorized three-wheeler type. The motorized three-wheeler and heavy vehicle types both have high-speed variance and low volume share in the traffic stream. Therefore, the 30-s sampling rate chosen for the observed density may present biased data for both. These two vehicle types are different in another aspect. The heavy vehicle type consists of buses that have regular frequency and therefore more likely to have uniform distribution. However, motorized three-wheelers are small vehicles. They filter through the standing traffic during the red indication and tend to come together in front of the traffic queue. At the green indication while other vehicles are accelerating and overtaking them, the motorized three-wheeler platoon disperses. Motorized three-wheelers have lower acceleration rates compared to passenger cars and buses and have a 50 km/h (30 mi/h) maximum speed due to their 150 cc engine size. This results in motorized three-wheelers staying in platoons longer to midblock than some other vehicle types.

Conclusion

Our experiment shows that the continuity equation is also valid under nonhomogeneous conditions. The basic difference between homogeneous and nonhomogeneous traffic is in the use of road space. For homogeneous traffic where car following and lane discipline behavior prevails, all traffic entities use an equal lane width. Traffic concentrations are in units of vehicles/km or vehicles/mi. However, in nonhomogeneous traffic, loose lane discipline prevails with no strict car-following logic. An adjustment to reflect nonhomogeneous traffic characteristics modified the original Wardrop equation. Each type of vehicle has an average concentration in the highway area. The concentration measurement is the average number of traffic entities of type j per unit area of highway, e.g., motorized two-wheelers/(km m) or motorized three-wheelers/(mi ft).

Quantifying nonhomogeneous traffic measures revealed several insights. The sum of densities of each traffic entity type comprising nonhomogeneous traffic on a given street area is the density of nonhomogeneous traffic in the same street area. The sum of every flow of traffic entity type measured across the width of the street area during 5 min is nonhomogeneous traffic flow given the same street width and duration. Finally, the space mean speed of nonhomogeneous traffic is the weighted harmonic speed of each traffic type's space mean speed.

The association between observation-based density and density derived using the continuity equation was moderate to strong. The correlation with zero intercept ranged from +0.50 to +0.90. The Wilcoxon signed-rank test compared observed and derived densities. At a 95%, confidence level, no significant difference existed between observed and derived density of light four-wheeler type, heavy vehicle type, motorized two-wheeler type, and nonmotorized two- and three-wheeler type, and cumulative of all, i.e., total density.

The results reduce the uncertainty many engineering practitioners have in using most macroscopic traffic models developed where homogeneous traffic prevails to solve nonhomogeneous traffic operational and design problems. With proper modification of concentration measure to the average number of traffic entities of type j per unit area of highway and other relevant adjustments such as passenger car units and friction factors, these operational models can be applicable.

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