Macroscopic relations of urban traffic variables: Bifurcations, multivaluedness and instability

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Abstract

Recent experimental work has shown that the average flow and average density within certain urban networks are related by a unique, reproducible curve known as the Macroscopic Fundamental Diagram (MFD). For networks consisting of a single route this MFD can be predicted analytically; but when the networks consist of multiple overlapping routes experience shows that the flows observed in congestion for a given density are less than those one would predict if the routes were homogeneously congested and did not overlap. These types of networks also tend to jam at densities that are only a fraction of their routes' average jam density.

This paper provides an explanation for these phenomena. It shows that, even for perfectly homogeneous networks with spatially uniform travel patterns, symmetric equilibrium patterns with equal flows and densities across all links are unstable if the average network density is sufficiently high. Instead, the stable equilibrium patterns are asymmetric. For this reason the networks jam at lower densities and exhibit lower flows than one would predict if traffic was evenly distributed.

Analysis of small idealized networks that can be treated as simple dynamical systems shows that these networks undergo a bifurcation at a network-specific critical density such that for lower densities the MFDs have predictably high flows and are univalued, and for higher densities the order breaks down. Microsimulations show that this bifurcation also manifests itself in large symmetric networks. In this case though, the bifurcation is more pernicious: once the network density exceeds the critical value, the stable state is one of complete gridlock with zero flow. It is therefore important to ensure in real-world applications that a network's density never be allowed to approach this critical value.

Fortunately, analysis shows that the bifurcation's critical density increases considerably if some of the drivers choose their routes adaptively in response to traffic conditions. So far, for networks with adaptive drivers, bifurcations have only been observed in simulations, but not (yet) in real life. This could be because real drivers are more adaptive than simulated drivers and/or because the observed real networks were not sufficiently congested.

Keywords: Urban mobility; Traffic congestion; Macroscopic fundamental diagram